Axionic wormholes with massive dilatons

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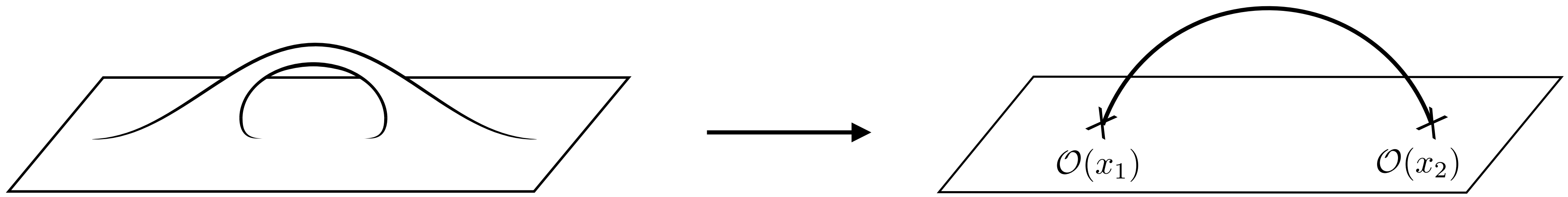
Work with S. Andriolo, G. Shiu, T. Van Riet - 2205.01119
(+ A. Hebecker, T. Mikhail - 1807.00824)

String Pheno '22 - Liverpool

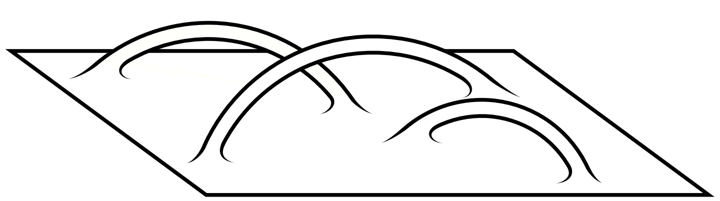
How do non-trivial topologies affect the Euclidean path integral of Quantum Gravity at low energies?

Wormholes & α-parameters

- In the IR, wormholes look like instanton anti-instanton pairs which couple to local operators $\mathcal{O}(x)$:



- In the dilute gas approximation, their contribution can be exponentiated, leading to a **bi-local effective action**:



$N = 3$

$$Z = \int [\mathcal{D}\Phi]\, e^{-S_0[\Phi]} \sum_{N=0}^{\infty} \frac{1}{N!} \left(e^{-S_{wh}} \int dx_1 \mathcal{O}(x_1) \int dx_1 \mathcal{O}(x_2) \right)^N$$

$$= \int [\mathcal{D}\Phi]\, e^{-S_0[\Phi]} \exp\left(e^{-S_{wh}} \int_{x_1} \int_{x_2} \mathcal{O}(x_1)\mathcal{O}(x_2) \right)$$

Coleman ’88, Preskill ’89…

Wormholes & α-parameters

- A bi-local effective action can be recast in a local form with the help of α**-parameters**:

$$e^{-\Delta S} \sim \exp\left(e^{-S_{wh}^{(0)}} \int_{x_1}\int_{x_2} \mathcal{O}(x_1)\mathcal{O}(x_2)\right) \sim \int d\alpha \, \exp\left(-\alpha^2 + \alpha\, e^{-S_{wh}^{(0)}/2}\int_x \mathcal{O}(x)\right)$$

Euclidean wormholes induce non-perturbative corrections to the effective action with **random couplings.**

$$Z \longrightarrow \int [\mathscr{D}\Phi]\, d\alpha\, e^{-\alpha^2} \exp\left(-\int_x \mathscr{L}_0[\Phi] + \alpha\, e^{-S_{wh}/2}\mathscr{O}(x)\right)$$

Coleman ’88, Preskill ’89…

Wormholes & α-parameters

- Non-perturbative wormhole effects are at the same time phenomenologically interesting and conceptually puzzling

Axionic wormholes generate non-perturbative potentials very relevant for particle physics and cosmology

$$\Delta S_{eff} = \alpha\, e^{-S_{wh}} \cos(\phi/f)$$

The presence of α-parameters is in tension with some basic swampland principle (e.g. no free parameters in QG)

McNamara, Vafa '20

α-parameters are also puzzling from the CFT perspective in standard holographic setups (in d>3)

Bergshoeff et al. '04 '05; Maldacena, Maoz '04; Arkani-Hamed, Orgera, Polchinski '07; Hertog, Trigiante, Van Riet '17

see Hebecker, Mikhail, PS '18 for review

Are wormhole effects real (in d>3)?

- **No:** (α-parameters are too weird)

  - Wormhole configurations represent gauge redundancies of QG

    McNamara, Vafa '20
    (c.f. Marolf, Maxfield '20)

  - Wormhole saddles are perturbatively 'unstable'

    See Gary's talk

  - Wormholes are non-perturbatively 'unstable' (WGC)

- **Yes:** (embrace the α-parameter)

  - Could be dual to ensemble averages of CFTs

    c.f. 2d (SYK) models:
    Saad, Shenker, Stanford '19;...
    Marolf, Maxfield '20;...

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The axionic WGC

$$\frac{S_{inst}\, f}{q} \lesssim M_P$$

$M_P = 1$

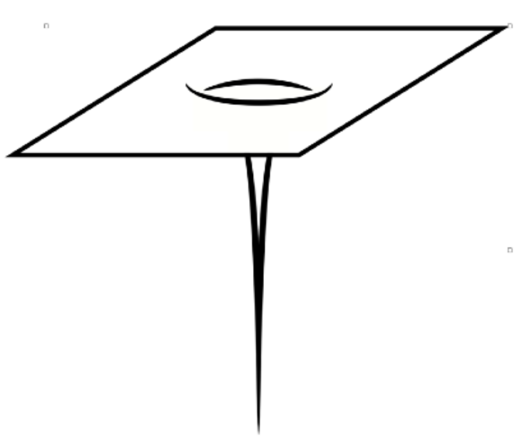
WGC for axions/2-forms

$$S_E = \int d^4x \sqrt{g} \left(-\frac{1}{2}\mathscr{R} + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2f^2}\, e^{-\beta\phi}\, H_3^2 \right)$$

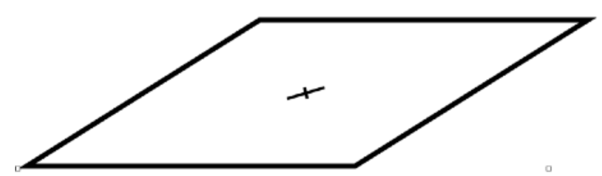
- Three types of O(4) symmetric solutions possible:

$$ds^2 = \left(1 + k\frac{a_0^4}{r^4}\right)^{-1} dr^2 + r^2 d\Omega_3^2\,, \qquad H = \frac{q}{2\pi^2}\, dVol_{S^3}\,, \qquad \phi = \phi(r)$$

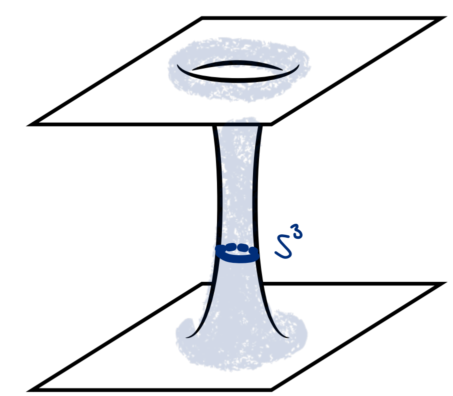
Cored instantons
$(k = +1)$



Flat (extremal) inst.
$(k = 0)$



Axion wormhole
$(k = -1)$



E. Bergshoeff, A. Collinucci, U. Gran, D. Roest and S. Vandoren, '04

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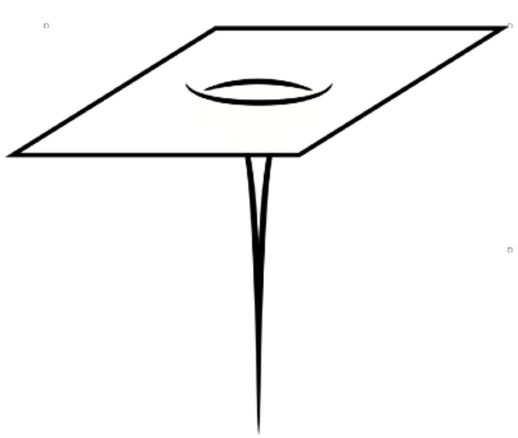
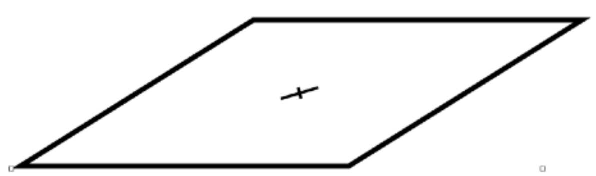
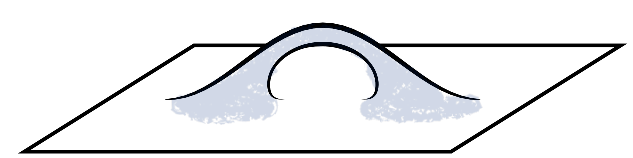






E. Bergshoeff, A. Collinucci, U. Gran, D. Roest and S. Vandoren, ‘04

WGC for axions/2-forms

- For $\frac{4}{\sqrt{6}} \equiv \beta_c \leq \beta$:

$M_P = 1$

WGC for axions/2-forms

- For $\frac{4}{\sqrt{6}} \equiv \beta_c \leq \beta$:

Only **cored** & **flat** instantons exist.

Related by compactification to **sub-extremal** & **extremal** BHs

eWGC (for axions) can be then inferred as:

$$S_{inst} \leq S_{flat} = \frac{2q}{\beta f}$$

Arkani-Hamed, Motl, Nicolis, Vafa '06;
Brown, Cottrell, Shiu, PS '15
Heidenreich, Reece, Rudelius '15

Flat (extremal) instantons have characteristic size: $e^{\beta\phi} = \left(1 + \frac{q\beta}{8\pi^2 f r^2}\right)^2$

mWGC (for 2-forms): extremal $q_m = 1$ should be microscopic

$$\Lambda \lesssim R_{q=1}^{-1} = \sqrt{\frac{8\pi^2 f}{\beta}}$$

$M_P = 1$

WGC for axions/2-forms

- For $\beta \leq \beta_c \equiv \frac{4}{\sqrt{6}}$:

$M_P = 1$

WGC for axions/2-forms

- For $\beta \leq \beta_c \equiv \frac{4}{\sqrt{6}}$:
  - Three solutions: **cored** & **flat** instantons and **wormholes** coexist.
  - No direct connection to black holes available.
  - Wormholes have larger charge-to-action ratio than flat instantons

Montero, Uranga, Valenzuela '15;

$$\frac{S_{wh}^{(Q)} f}{Q} < \frac{2 S_{flat}^{(Q)} f}{Q} = \frac{4}{\beta}$$

WGC?

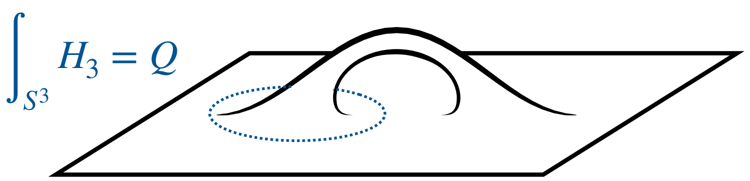
  - Seems more reasonable to demand that there exist small wormholes (or instantons) with charge-to-action ratio s.t.

$$\frac{S_{wh}^{(q)} f}{q} \leq \frac{S_{wh}^{(Q)} f}{Q} = \frac{4}{\beta} \sin\left(\frac{\pi}{2} \frac{\beta}{\beta_c}\right)$$

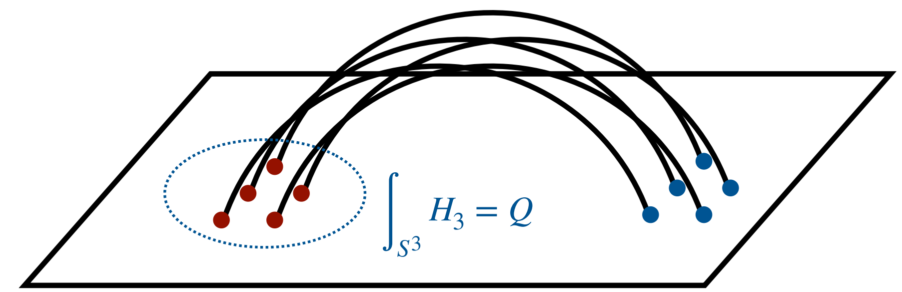
Hebecker, Mangat, Theisen, Witkowski '16
Hebecker, Mikhail, PS '18

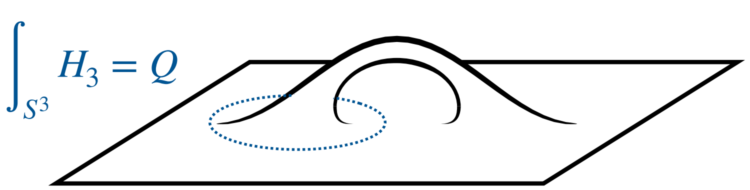
WGC for axions/2-forms

- Electric 0-form WGC (for $\beta < \beta_c$):

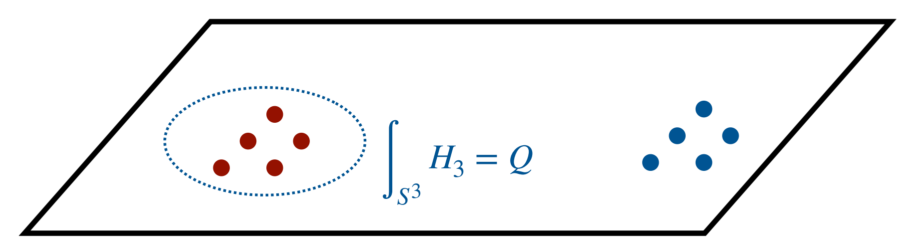


$$\frac{S_{wh}^{Q}}{Q} \geq \frac{S_{wh}^{(q)}}{q}$$





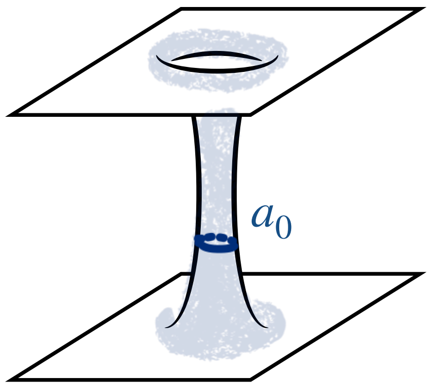
$$\frac{S_{wh}^{Q}}{Q} \geq \frac{2S_{inst}^{(q)}}{q}$$



(No α–parameters)

$M_P = 1$

WGC for axions/2-forms



- Magnetic 2-form WGC (for $\beta < \beta_c$):

Wormholes have an associated (neck) length scale:

$$a_0^4 = \frac{q^2}{24\pi^2 f^2} \cos\left(\frac{\pi}{2}\frac{\beta}{\beta_c}\right)$$

Demanding that wormholes with $q = 1$ are microscopic:

$$\Lambda^2 \lesssim a_0^{-2}\Big|_{q=1} \sim \frac{f}{\cos\left(\frac{\pi}{2}\frac{\beta}{\beta_c}\right)}$$

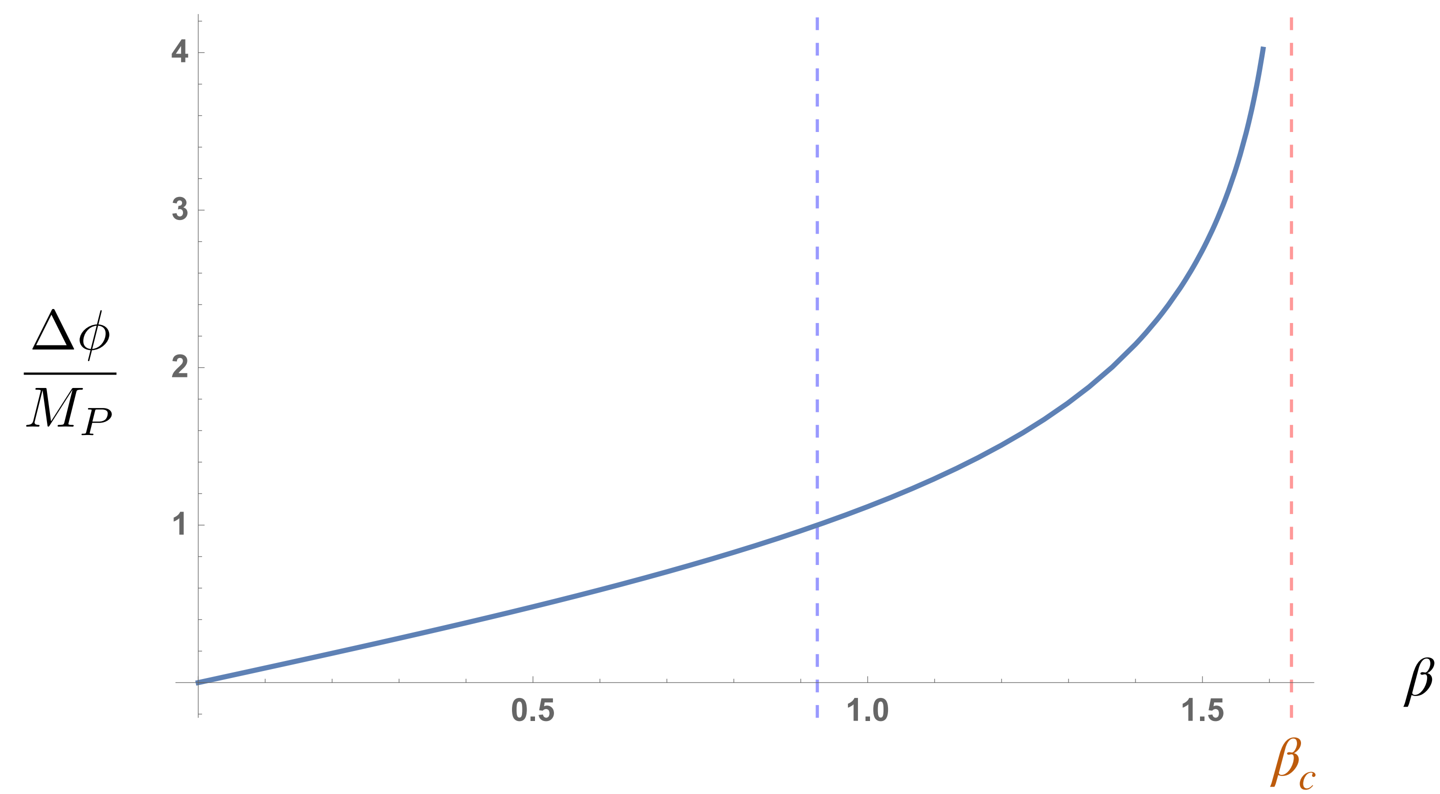
For $\beta = 0$ (no dilaton) this had been motivated by demanding consistency of evaporation of axionic black holes.

Hebecker, PS '17

WGC for axions/2-forms

- Wormholes and large distances (for $\beta < \beta_c$):

$$e^{\beta\,\Delta\phi} = \cos^{-2}\left(\frac{\pi}{2}\frac{\beta}{\beta_c}\right)$$



Dilaton displacement is transplackian for $\beta \gtrsim 1$

Hebecker, Mangat, Theisen, Witkowski '16

Axionic wormholes with massive dilatons

Andriolo, Shiu, PS, Van Riet ‘22

Wormholes with massive dilaton

$M_P = 1$

$q > 0$

- We study wormhole solutions when we include a dilaton mass

$$S_E = \int d^4x \sqrt{g} \left[-\frac{R}{2} + \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2f^2} e^{-\beta\phi} H^2 + m^2\phi^2 \right]$$

- Study wormhole solutions as a function of $\frac{m^2 q}{f} \approx$ (dilaton mass x neck radius)^2
  - Large wh. $\frac{m^2 q}{f} \gg 1$: dilaton freezes and decouples (same as $\beta \to 0$)
  - Small wh. $\frac{m^2 q}{f} \ll 1$: dilaton is effectively massless

Wormholes & axionic WGC

$M_P = 1$
$q > 0$

- Our main interest is in the action-to-charge ratio of wormholes

$$s_q \equiv \frac{S_q f}{q}$$

  - For large wormholes ($\frac{m^2 q}{f} \gg 1$) perturbative expansion yields:

$$s_q = \sqrt{\frac{3}{2}}\pi^3\left(1 - 4\pi\sqrt{3}\,\frac{\beta^2 f M_P}{q\, m^2}\right) + \ldots$$

c.f. Andriolo, Huang, Noumi, Ooguri, Shiu ‘20

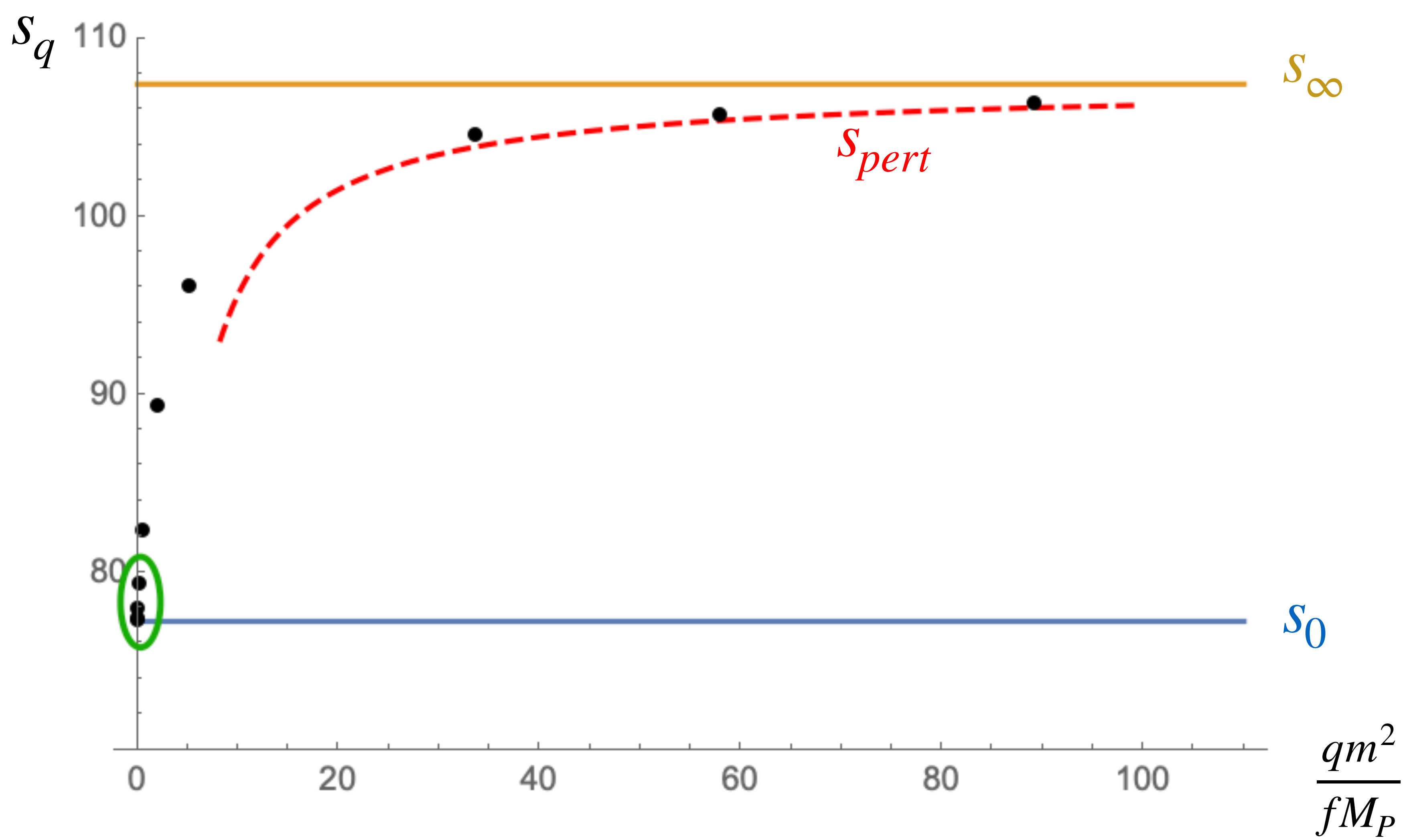
  - For small wormholes ($\frac{m^2 q}{f} \lesssim \mathcal{O}(1)$) we compute numerically

c.f. Kallosh, Linde, Linde, Susskind ’95

Wormholes & axionic WGC

$M_P = 1$
$q > 0$
$\beta = \sqrt{2}$

- Action-to-charge ratio: $s_0 < s_q < s_\infty$



  - This confirms the "mild-WGC": larger wormhole have bigger action-to-charge ratio, and can hence "decay" into smaller ones

Andriolo, Shiu, PS, Van Riet '22

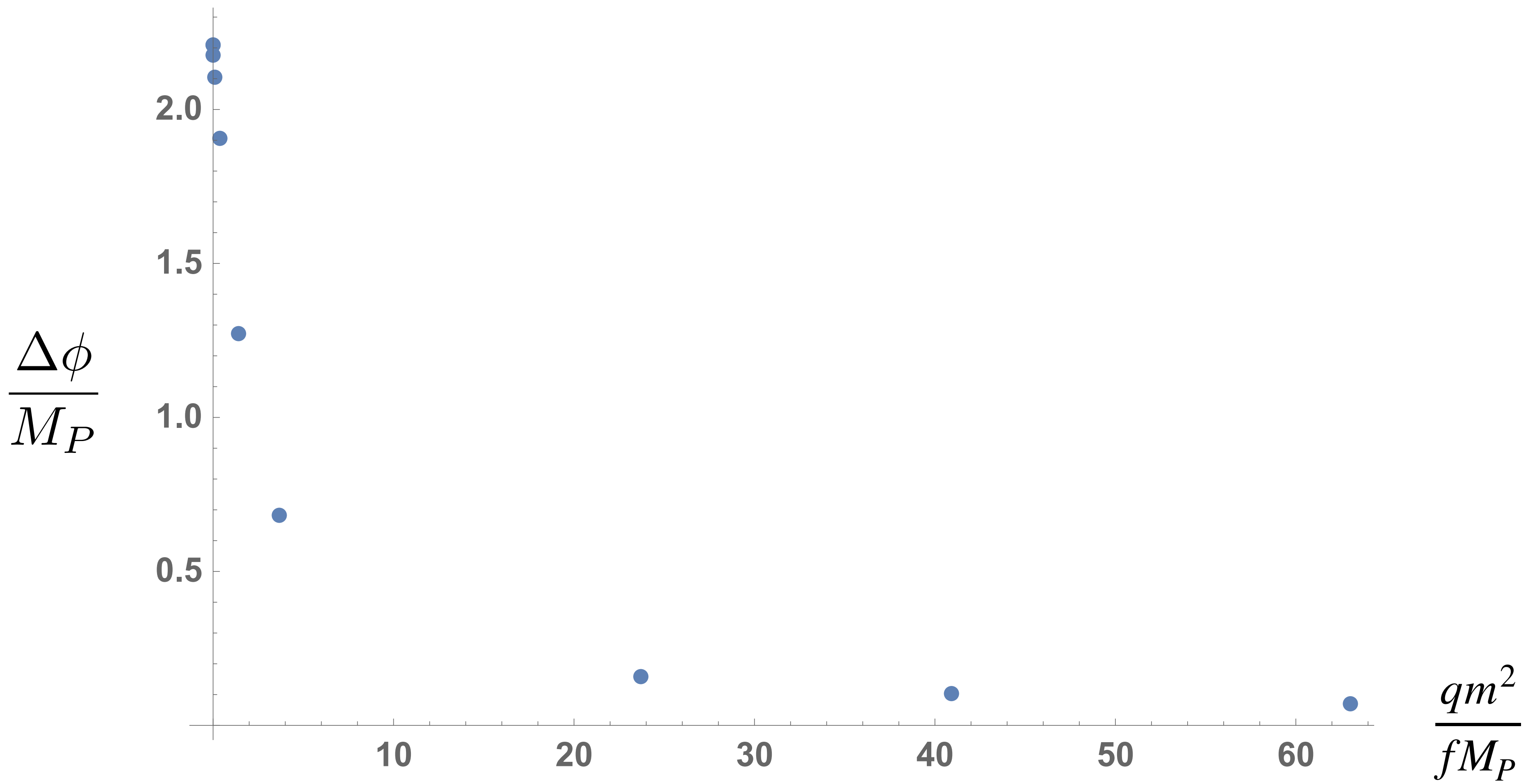
Wormholes & axionic WGC

$M_P = 1$

$q > 0$

$\beta = \sqrt{2}$

- Dilaton displacement:



- The dilaton mass improves the dilaton behaviour of large whs

Andriolo, Shiu, PS, Van Riet '22

Conclusions

Conclusions

- Wormholes and instantons are crucial ingredients in axion pheno.
- Space of solutions depends crucially on dilaton parameter β
  - For $\beta \geq \beta_c$ wormholes do not exist: WGC well understood.
  - For $\beta < \beta_c$ wormholes exist: WGC becomes obscure.
- Wormholes with massive dilatons are phenomenologically relevant, and effectively interpolate between $0 \leq \beta < \beta_c$:
  - The mild (wormhole) WGC is satisfied: $S^{(q)}_{wh}/q \leq S^{(Q)}_{wh}/Q$
  - Ultimate fate of wormholes & α-parameters still unclear.
- Task: clean string models with $\beta < \beta_c$ and/or dilaton mass
  - Explore wormhole extremality/stability & holographic avatars

스트링 페노 23

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대전광역시 2023년 07월 3-7일

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String Pheno 23

Center for Theoretical Physics of the Universe,
Institute for Basic Science,
Daejeon, South Korea, 3-7 July 2023

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